# An Efficient Decomposed Method in Harmonic Domain For Solving Nonlinear Time-Periodic Magnetic Problems

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**A new method is presented to calculate the nonlinear time-periodic magnetic field coupled with electric circuits. Periodic excitations and unknown magnetic vector potentials in nonlinear magnetic field are represented by Fourier-series, and the harmonic-balance equation is obtained by approximating the nonlinear medium parameters e.g. magnetic reluctivity in frequency domain. A new decomposed method is proposed to decouple harmonic solutions so that each harmonic component can be solved separately and in parallel. An example is used to verify the effectiveness and efficiency of the proposed method in 2-D problems.**

***Index Terms*—Fourier series, harmonic solutions, magnetic reluctivity, time-periodic.**

## I. Introduction

THE nonlinear time-periodic eddy-current problems are usually solved by time-stepping method, which may require a lot of periods to achieve the periodic steady state. Apparently the computational cost is expensive when the nonlinear dynamic electromagnetic problems with periodic excitation are computed in time domain.

Due to the periodic characteristic of solutions, some new methods have been presented and developed in recent years. The time-periodic method is presented to compute nonlinear eddy current problems [1], however, non-symmetric coefficient matrix is not a good choice for large-scale problems in numerical computation. The periodic field quantities are approximated by triangular series to obtain the harmonic-balanced system equation for computation of nonlinear magnetic field in frequency domain [2-3], but a major drawback is that the size of stiffness matrix depends on the number of harmonics and may lead to increasing computational cost in large-scale computation.

The fixed-point technique has been widely used in time-stepping method and harmonic-balanced method to compute nonlinear eddy current problems [4-6]. By introducing the fixed-point reluctivity or permeability the nonlinear relationship ***B***-***H*** can be split into two parts, i.e., the linear part and nonlinear part correspond to the time-independent stiff matrix and the right-hand side respectively [7]. As a result, the fast Fourier transformation (FFT) can be used and harmonic solutions are decoupled and solved iteratively. A key procedure of the fixed-point harmonic-balanced method is the optimal determination of the fixed-point reluctivity or permeability which plays an important role in convergent performance of harmonic solutions. Different strategies have been presented to achieve fast and stable convergence of solutions [8-9].

In this paper, the harmonic-balanced system equations are established based on harmonic balance theory, in which the magnetic reluctivity $\nu$ is approximated by Fourier series. Thus, a stiffness matrix related to dc component of reluctivity ($\nu_0$) can be generated with the other components brought to right-hand side. Finally harmonic solutions can be solved separately in frequency domain.

## II. Decomposition in Frequency Domain

The two-dimensional nonlinear magnetic field involving eddy current can be formulated by using the magnetic vector potential $\boldsymbol{A}$,

$$\nabla\times\nu\nabla\times\boldsymbol{A}+\sigma\frac{\partial\boldsymbol{A}}{\partial t}=\boldsymbol{J} \tag{1}$$

where $\sigma$ is the conductivity and $\boldsymbol{J}$ is the impressed current density.

Due to the time periodicity of field quantities, any vectorial or scalar periodic variables such as $\boldsymbol{x}(t)$ can be represented by complex Fourier series,

$$\boldsymbol{x}(t)=\sum_{n=-N_h}^{N_h}\boldsymbol{X}_n e^{jn\omega t} \tag{2}$$

where $N_\mathrm{h}$ is the total number of harmonics truncated in computation and $\omega$ is the angular frequency.

Galerkin weighted-residual method applied to (1) over the whole computational region leads to the following ordinary differential equation system

$$\boldsymbol{M}(\sigma)\frac{dx(t)}{dt}+\boldsymbol{S}(\nu)x(t)=f \tag{3}$$

where $\boldsymbol{M}$ and $\boldsymbol{S}$ are, respectively, the mass matrix and the stiffness matrix.

Consequently the harmonic-balanced system equations can be obtained from (3) after applying the Fourier transform,

$$j\omega\boldsymbol{M}(\sigma,\boldsymbol{N})\boldsymbol{X}+\boldsymbol{S}(\boldsymbol{D})\boldsymbol{X}=\boldsymbol{F} \tag{4}$$

where $\boldsymbol{N}$ and $\boldsymbol{D}$ are, respectively, the harmonic matrix and the reluctivity matrix [2].

Finally (4) can be decomposed to obtain a new harmonic-balanced matrix equation considering external circuits coupled with magnetic field,

$$\begin{bmatrix} jn\omega\boldsymbol{M}(\sigma)+\boldsymbol{S}(\nu_0) & \boldsymbol{G}_n \\ \boldsymbol{C}_n & \boldsymbol{Z}_n \end{bmatrix}\begin{Bmatrix}\boldsymbol{A}_n\\ \boldsymbol{J}_n\end{Bmatrix}=\begin{Bmatrix}\boldsymbol{P}_n\\ \boldsymbol{U}_n\end{Bmatrix}(n=0,\ldots,N_h) \tag{5}$$

where $\boldsymbol{A}_n$ is the n-th harmonic solution of vector potential, $\boldsymbol{J}_n$ is the n-th harmonic of the impressed current density, $\boldsymbol{C}$ represents the circuit-field coupled matrix and $\boldsymbol{Z}$ is the corresponding impedance matrix. $\boldsymbol{P}_n$ is related to convolution product of vector potential and reluctivity in frquency domain, and $\boldsymbol{U}_n$ is the n-th harmonic component of the driving voltage.

It is noticed that the system equation in (5) is very similar with that in the fixed point method, and the main difference between them is the choice of fixed point reluctivity [9].

## III. Computation Example

The 2-D problem consists of exciting coils and a laminated iron core. Fig. 1 shows one quarter of the whole model, in which there are two coils in series on the laminated core. The exciting coil is connected to a voltage source of 50 Hz. The model is magnetized by a dc-biased excitation in experiment so that the laminated core is saturated significantly.

Fig.2 depicts the comparison between numerical and experimental data. It can be seen that the computed exciting current by the proposed method and that by the fixed point method agrees well with each other [9]. The comparison between the computed results and the measured one shows consistency, which verifies the effectiveness of the newly proposed method.

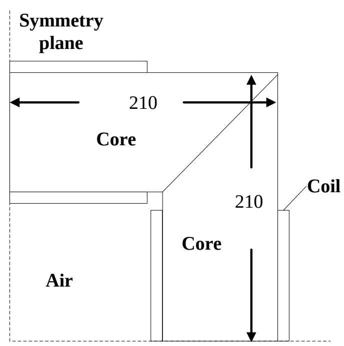


Fig. 1 Geometric structure of the laminated core model (unit: mm)

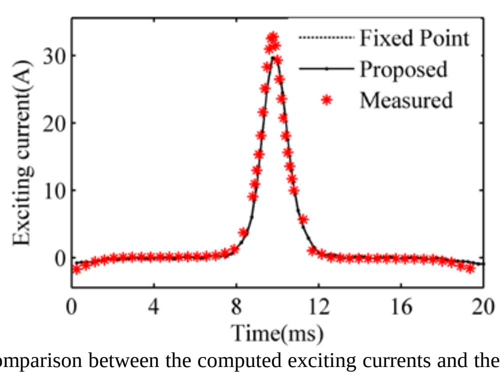


Fig. 2 Comparison between the computed exciting currents and the measured result ($U$=504V/m, $I_{dc}$=2.37A) .

The computational results are also shown in Fig.3 where the evolution of the maximum variation of the reluctivity is given. It can be seen that the proposed method can guarantee a high rate of convergence of harmonic solutions, even though the amplitude of the applied voltage is 504 volts and the dc current is 2.37 amperes, which means a significant saturation of the iron core.

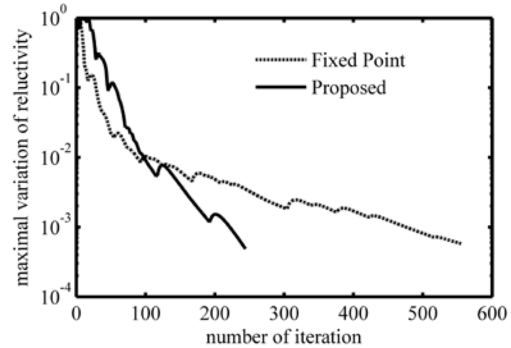


Fig. 3 Maximum variation of reluctivity in computation by the proposed method and the fixed point method.

## IV. Conclusion

A new efficient method independent of fixed-point reluctivity has been presented to compute the nonlinear time-periodic magnetic problems. The decomposed harmonic-balance equation provides the applicability of parallel computing based on decoupling of harmonic solutions, which makes it possible to efficiently solve nonlinear problems under multi-harmonic excitations. Comparison of the convergent performance between the proposed method and the fixed point method will be presented and analyzed in detail in the extended paper when eddy currents exist in the nonlinear magnetic problems.

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